MATH 1105 - FALL 2008 - 10-17-08
SECTION 2
IN-CLASS PROBLEMS

(1) A committee of 3 people is chosen out of 10 people.

a. How many ways are there to choose this committee?
$\binom{10}{3}$

b. How many ways are there to choose the people that are not in the committee?
$\binom{10}{7}$

c. How are your answers from part a and b related? Why? Check this using the formula for choose notation.
The answers are equal because choosing which of the ten people are in the committee is the same as selecting which of the 10 are not in the committee.
$\binom{10}{3} = \frac{10!}{3!7!} = \binom{10}{7}$.

d. Suppose $0 \leq k \leq n$ and that a committee of k people is chosen from n people. Repeat parts a, b, and c for this situation. What is the general formula that this gives you? Check this using the formula for choose notation.
a. $\binom{n}{k}$
b. $\binom{n}{n-k}$
c. The answers are equal because choosing which of the n people are in the committee is the same as selecting which of the n are not in the committee.
$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$

(2) A committee of 3 people is chosen out of 5 people.

a. How many ways are there to choose this committee?
$\binom{5}{3}$

b. Sam is one of the five people. How many of these possible committees is Sam on?
$\binom{4}{2}$

c. How many of these committees exclude Sam?
$\binom{4}{3}$

d. How are your answers from parts a, b, and c related. Check this formula using choose notation.
Since each choice of a committee either has Sam on it or does not have Sam on it, the sum of the answers from parts b and c most be the answer from part a. So $\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$.
$\binom{4}{2} + \binom{4}{3} = \frac{4!}{2!2!} + \frac{4!}{3!1!} = \frac{4!3}{3!2!} + \frac{4!2}{3!2!} = \frac{4!(3+2)}{3!2!} = \frac{5!}{3!2!}$

e. Suppose $n \geq 1$ and $0 \leq k \leq n$ and that a committee of k people is chosen from n people. Assume that Sam is one of these n people and repeat parts a, b, c, and for this situation. What is the general formula that this gives you? Check this using the formula for choose notation.
a. $\binom{n}{k}$
b. $\binom{n-1}{k-1}$
c. $\binom{n-1}{k}$
d. Since each choice of a committee either has Sam on it or does not have Sam on it, the sum of the answers from parts b and c most be the answer from part a. So $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-k)!(k-1)!} + \frac{(n-1)!}{(n-k+1)!k!} = \frac{(n-1)!k}{(n-k)!k!} + \frac{(n-1)!(n-k)}{(n-k)!k!} = \frac{(n-1)!(k+n-k)}{(n-k)!k!} = \frac{n!}{(n-k)!k!}$

(3) Consider the algebraic expression $(a+b)^3$.

a. Write this expression in the form $\alpha a^3 + \beta a^2 b + \gamma a b^2 + \delta b^3$, where α,β,γ, and δ are constants. ie, What numbers are α, β, γ and δ?
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, $\alpha = 1$, $\beta = 3$, $\gamma = 3$, and $\delta = 1$.

b. Check that $(a+b)^3 = aaa + aab + aba + abb + baa + bab + bba + bbb$.
True by the distributive law of multiplication.

c. Check that your answer for β in part a is the same as the number sets of orders of three total a's and b's that has 2 a's and 1 b. A good way to see this is to look at the equation given in part b.
There are three terms in list in part b that have two a's and one b, and indeed $\beta = 3$.

d. Suppose that an ordered string of three a's and b's has one b. How many different strings are there with one b? What does part c tell you about this number

in relation to β.
Since the set of orders of the letters a and b that have two a's and one b's is determined by the position of the b and there are $\binom{3}{1}$ positions for the b, there are 3 sets orders of a's and b's with two a's and one b.

e. Suppose that $0 \leq k \leq n$ and an ordered string of n a's and b's has k b's. How many different strings are there with k b's?
Since of the n positions in the string, we must pick k of them to be the positions of b's, there are $\binom{n}{k}$ such strings.

f. What is the coefficent on the term $a^{n-k}b^k$ resulting from expanding the expression $(a+b)^n$?
The number of the strings counted in part e is this coefficient because if you write out of the length n strings of a and b we will add 1 to the coefficient for each such string. Hence the coefficient is $\binom{n}{k}$.